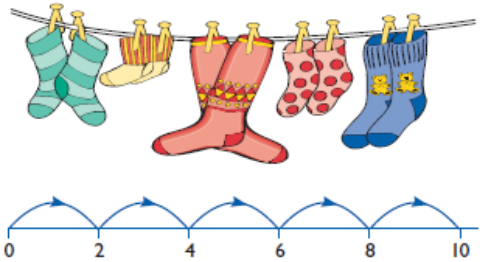
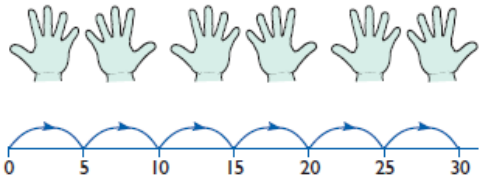
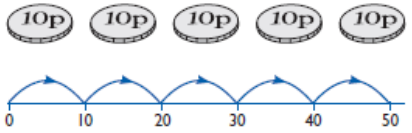
Key representations to support conceptual understanding of multiplication and division



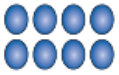
$2 + 2 + 2 + 2 + 2 = 10$
$2 \times 5 = 10$
2 multiplied by 5
5 pairs
5 hops of 2



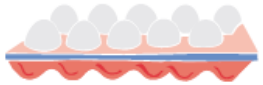
$5 + 5 + 5 + 5 + 5 + 5 = 30$
$5 \times 6 = 30$
5 multiplied by 6
6 groups of 5
6 hops of 5



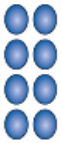
$10p + 10p + 10p + 10p + 10p = 50p$
$10p \times 5 = 50p$
5 hops of 10



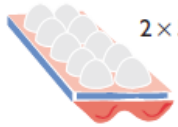
$4 \times 2 = 8$
$2 \times 4 = 8$



$5 \times 2 = 10$
$2 \times 5 = 10$

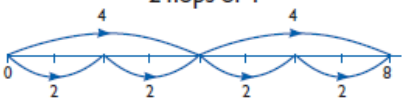


$2 \times 4 = 8$
$4 \times 2 = 8$



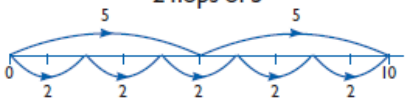
$2 \times 5 = 10$
$5 \times 2 = 10$

2 hops of 4

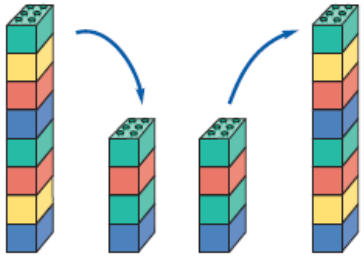


4 hops of 2

2 hops of 5

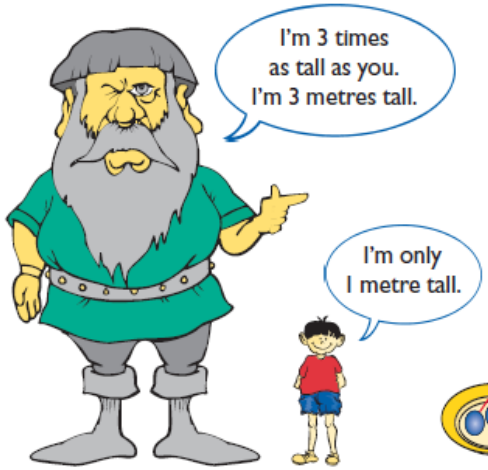


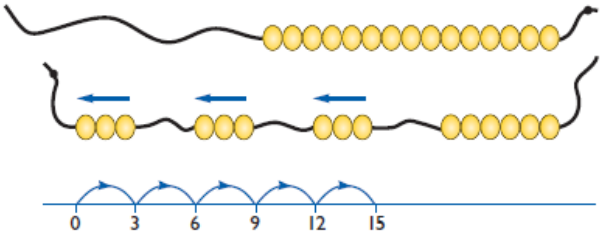
5 hops of 2



half of 8 is 4
$8 \div 2 = 4$

double 4 is 8
$4 \times 2 = 8$

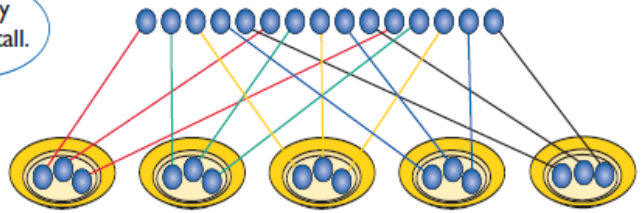


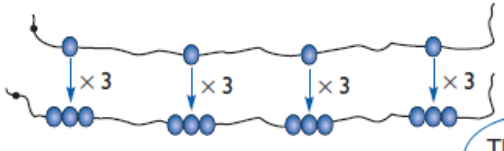


How many 3s in 15?

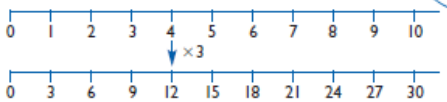
$15 \div 3 = 5$

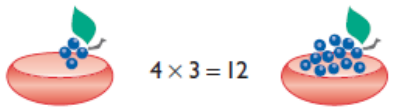
5 hops in 15. How big is each hop?
$15 \div 5 = 3$
15 shared between 5



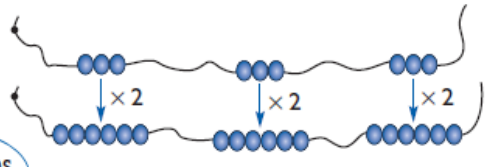


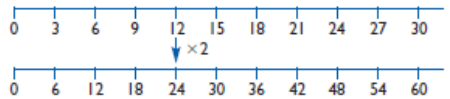
Three times as many





$4 \times 3 = 12$



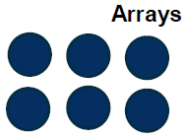
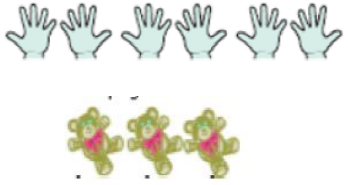
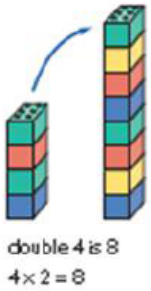
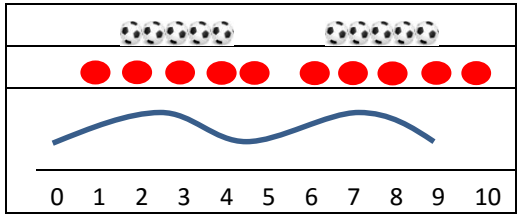
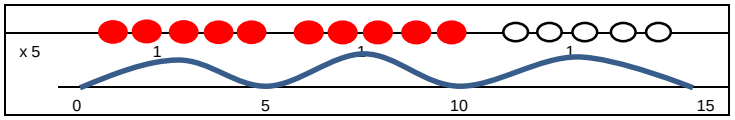
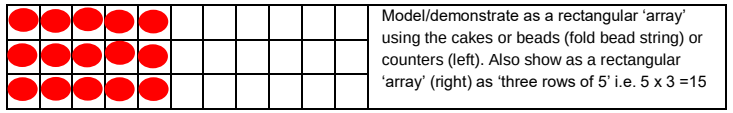
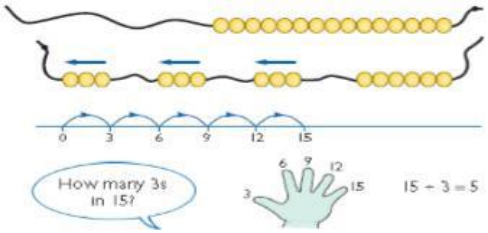
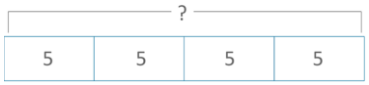


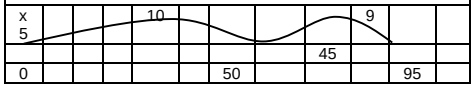
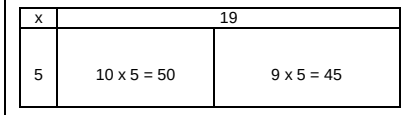
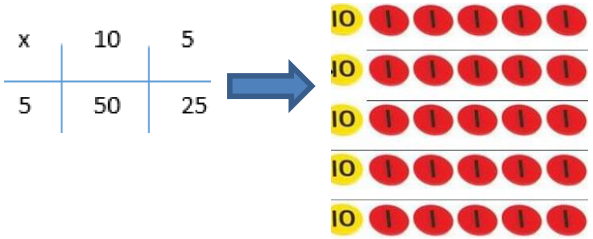
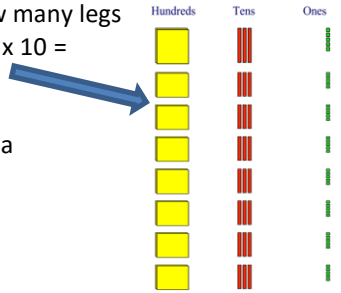
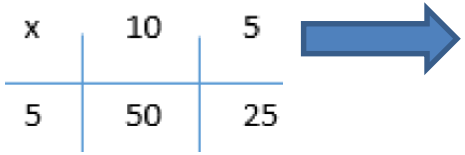
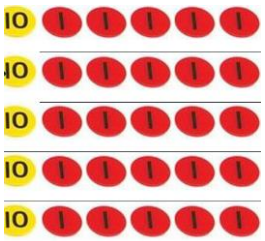
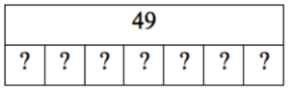
Twice as many

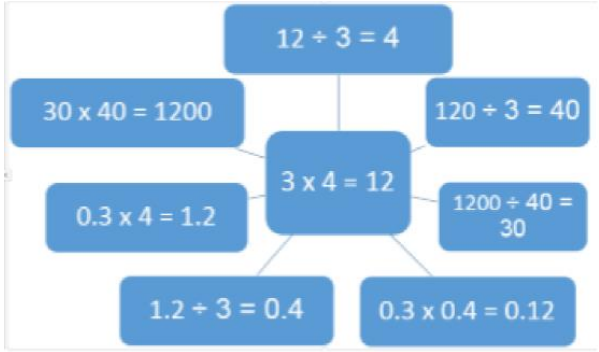
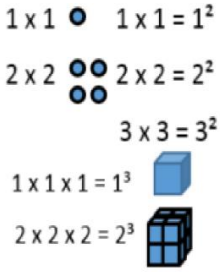
$12 \times 2 = 24$

Multiplication

	Nursery multiplication	Reception multiplication
Mental/ Written	How many feet have these three teddy bears got altogether?  Strategy: begin to recognise repetitive addition of groups of the same size Recording: teacher demonstration of appropriate pictorial recording where appropriate. Vocabulary: groups, sets Equipment: everyday objects, counters, fingers **n.b. signs for multiplication and division are used for adult understanding in this year group as they are not introduced until year two**	How many wheels do we need for these three lego cars?  2 2 2 2 2 2 Strategy: begin to recognise repetitive addition of groups of the same size; counting in steps of 10 or 2 Recording: teacher demonstration of calculation to match pictorial recording using standard notation of + and =. Demonstrate on number line. Vocabulary: double, groups of, sets of, lots of Equipment: everyday objects: e.g. cars, chairs, bears, children, fingers, gloves, toy cars, pairs of socks. Also 'maths' objects e.g. counters, **n.b. signs for multiplication and division are used for adult understanding in this year group as they are not introduced until year two**

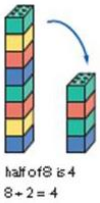
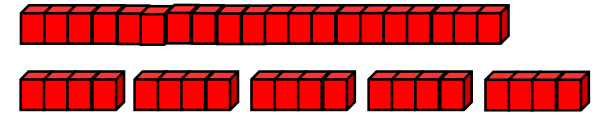
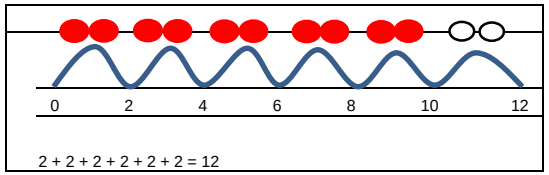
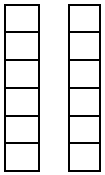
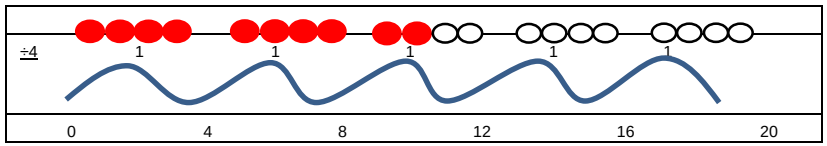
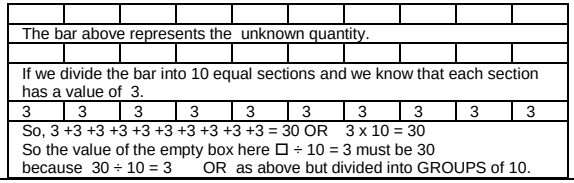
	Year 1	Year 2
Mental	Use of visual models to support counting in 2, 5, 10 Ensure children begin to see the patterns of counting in 2, 5, 10. Halving numbers up to: 10 + 10 = 10 x 2 20 –10 = 20 ÷ 2 Children do not need to record number sentences using the symbols. Develop the vocabulary by encouraging children to explain what they are doing. Share 12 sweets between two children. How many do they each have?	Count in steps of 2, 3, and5 from 0, and in tens from any number, forward or backward Use knowledge of doubling: 2 x 5 = 10, 5 x 2 = 10, 2 x 10 = 20, 10 x 2 = 20
Written	    Ben had 5 football stickers on each page. He has two pages of stickers. How many does he have altogether?  Show two groups of 5 football stickers. Model this on a bead bar/string. Demonstrate recording of two groups of 5 stickers on a number line. Also show as a rectangular ‘array’ i.e. two groups of 5 or 5 + 5 = 10 **n.b. signs for multiplication and division are used for adult understanding in this year group as they are not introduced until year two**	Anna has 3 boxes of cakes. Each box contains 5 cakes. How many cakes does she have altogether? Show how you worked this out. For example, ‘**practically**’ and counting in 5s.  Or written as: 3 x 5 = 15, 15 ÷ 5 = 3 This may also be modelled on a bead bar/string as ‘repeated’ addition i.e. 5 + 5 + 5= 15 AND ‘5 three times equals 15’ so 5 x 3 = 15 or ‘five multiplied or times by three equals 15’. Demonstrate recording on a number line:   Other models for multiplication:  

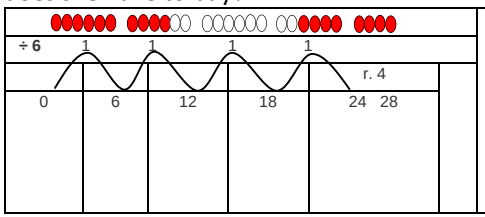
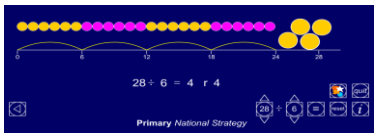
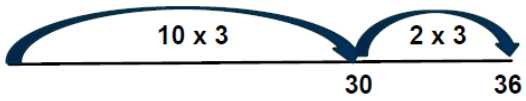
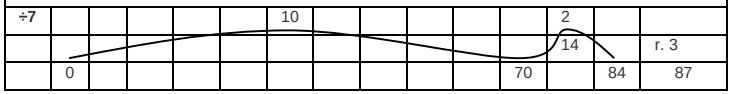
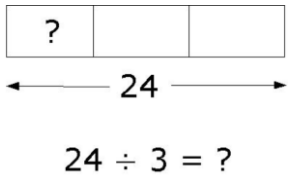
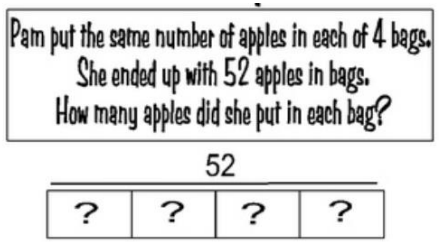
	Year 3	Year 4
Mental	If the children know 2/5/10 facts they now need to learn: 3/4/8 facts With corresponding division facts. Recall facts along with counting in steps sizes.  4 x 3 = 3 x 4 12 ÷ 3 = 4 12 ÷ 4 = 3	If the children know multiplication and division facts for: 2/5/10/3/4/8/ they now need to learn: 6/7/9/11 To solve 24 x 3, use knowledge of factor pairs. 8 x 3 x 3 6 x 4 x 3 In measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).
Written	Mark drives 19 miles to work every day. He does this on Mondays, Tuesdays, Wednesdays, Thursdays and Fridays. How many miles does he travel to work in one week? Demonstrate using a numberline as '**repeated' addition** using straws or Base 10 as a model: This can be rewritten as: 10 x 5 + 9 x 5 = 50 + 45 = 95  Leading to formal recording of 'grid multiplication' as a representation of the rectangular array. (Link with 'area') 50 + 45 = 95 so, 19 x 5 = 95   	The class wants to make 235 spiders for a display. How many legs do they need to make? 235 x 8 is approximately 235 x 10 = 2350  Leading to formal recording of '**grid multiplication'** as a representation of the rectangular array. x 200 30 5 8 1600 240 40 1600 + 240 + 40 = 1880 so, 235 x 8 = 1880 Leading to an expanded 'vertical' layout:   1 5 X 5 2 5 + 5 0 7 5 bar model➔ 

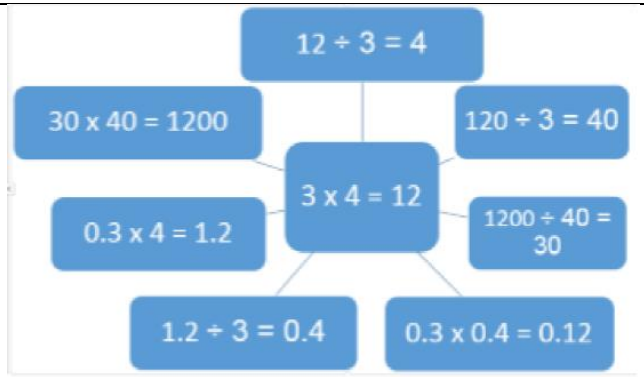
	Year 5	Year 6
Mental	  **Multiplying and dividing whole numbers and decimals by 10, 100 and 1000.** | Thousands | Hundreds | Tens | Ones | /10 (tenths) | /100 (Hundredths) | | | | | | | |	They undertake mental calculations with increasingly large numbers and more complex calculations. Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures. Pupils explore the order of operations using brackets; for example: 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9. Common factors can be related to finding equivalent fractions. Calculate 900 ÷ (45 × 4). A bag of four oranges costs thirty seven pence. How much do twelve oranges cost?
Written	Where possible, model using Base 10 equipment (see Y4 example) or using counters (see Y3 example). Use the equipment to show partitioning i.e. 56 x 27 = 50 x 20 + 50 x 7 + 6 x 20 + 6 x 7 = (50 + 6) x 27 **Short and Long Multiplication:** 2307 x 8 = Estimate: 2000 x 8 = 16000 Calculate: (Short multiplication) 2 3 0 7 X 8 18 4 5 6 2 5 1431 x 23 = Estimate: 1431 x 20 = 28620 1 Calculate: (Long multiplication) 1 4 3 1 X 2 3 4 2 9 3 (1431 x 3) 2 8 6 2 0 (1431 x 20) 3 2 9 1 3 1 1	Use the equipment e.g. Base 10 materials, to show partitioning i.e. 1328 x 43 = 1000 x 43 + 300 x 43 + 20 x 43 + 8 x 43 = (1000 + 300 + 20 + 8) x 43 Leading to recording using '**grid multiplication'** as a representation of a rectangular array. Possible use of Excel to generate array – see Y5 example. As **1328 x 43 = 43 x 1328** (commutative), the layout for grid multiplication can also be presented 'vertically' as a step towards 'long multiplication'. x | 40 | 3 1000 | 40 000 | 3000 300 | 12 000 | 900 20 | 800 | 60 8 | 320 | 24 53 120 + 3984 So, 53 120 + 3 984 = 57 104 Leading to a compact 'vertical' layout. Either begin with least or most significant figure. 1 3 2 8 x 4 3 3 9 8 4 5 3 1 2 0 5 7 1 0 4 1 1 3 mulitplied by 1328 is 3984 40 multiplied by 1328 is 53 120 As **18·6 x 4·4 = 4·4 x 18·6** (commutative), the layout for grid multiplication can also be presented 'vertically' as a step towards 'long multiplication'. x | 4 | 0·4 10 | 40·0 | 4·00 8 | 32·0 | 3·20 0.6 | 2·4 | 0·24 74·4 + 7·44 So, 74.40 + 7.44 = 81·84 Leading to an expanded 'vertical' layout. Either begin with least or most significant figure and then a compact 'vertical' layout – see below.

Division

	Nursery division	Reception division
Mental/ Written	If we share out these cakes so everyone has one each, how many will be left over? If everyone has two cakes, how many children will be able to have cakes today?  Strategy: begin to recognise sharing equally; also repetitive addition or subtraction of groups of the same size Recording: teacher demonstration of appropriate pictorial recording where appropriate. Vocabulary: groups, share Equipment: everyday objects, counters, fingers **n.b. signs for multiplication and division are used for adult understanding in this year group as they are not introduced until year two**	Can we share out these cakes fairly? How shall we do it? If we put two cakes on each plate, how many plates do we need?  Strategy: begin to recognise sharing equally; also repetitive addition of 'groups' of the same size i.e. grouping Recording: teacher demonstration of calculation to match pictorial recording using standard notation of + and =. Demonstrate on number line. Vocabulary: groups, share, left over, half Equipment: everyday objects, counters, fingers **n.b. signs for multiplication and division are used for adult understanding in this year group as they are not introduced until year two**

	Year 1 division	Year 2 division
Mental	Use of visual models to support counting in 2, 5, 10 Ensure children begin to see the patterns of counting in 2, 5, 10. Halving numbers up to: 10 + 10 = 10 x 2 20 –10 = 20 ÷ 2  Children do not need to record number sentences using the symbols. Develop the vocabulary by encouraging children to explain what they are doing. Share 12 sweets between two children. How many do they each have? Grouping and sharing 	How many sticks of 4 cubes can you make from 20 cubes? OR If 20 cubes are shared equally between 4 people, how many cubes do they each get?   Use of arrays, e.g. 8÷2=4, 8÷4=2 count in steps of 2, 3, and5 from 0, and in tens from any number, forward or backward Use knowledge of doubling and halving, e.g. 10 ÷ 2 = 5, 10 ÷ 5 = 2, 20 ÷ 2 = 10, 20 ÷ 10 = 2
Written	  EQUAL SHARING Show 12 sweets. Share the sweets equally between two i.e. one for you and one for me – we've used 2 of the sweets. Model this on a bead bar/string and demonstrate recording of one group of 2 sweets on a number line. *Another one for you and one for me – we've used 2 more of the sweets*. Model this on a bead bar/string and demonstrate recording of the second group of 2 sweets on a number line etc. ALSO: use as above with GROUPING e.g. We have 12 sweets. If each child has 2 sweets, how many children will have 2 sweets? i.e. two for you – that's one group of two – that's one person - model on bead bar and record on a number line as above. **n.b. signs for multiplication and division are used for adult understanding in this year group as they are not introduced until year two**	GROUPING Show 20 cubes. Divide the cubes into groups of 4 i.e. one group of 4, two groups of 4 and so on. Model this on a bead bar/string and demonstrate recording of one group of 4 cubes on a number line…another group of 4 cubes etc. So, 4 + 4 + 4 + 4 + 4 = 4 x 5 = 20 so 20 ÷ 4 = 5 i.e. 20 divided into groups of 4 equals 5.  EQUAL SHARING use same model as above. e.g. We have 20 cubes. Share the cubes equally between each person – we've used 4 of the cubes. Model this on a bead bar/string and demonstrate recording of one group of 4 cubes on a number line….. Another one for each person etc. So, 4 + 4 + 4 + 4 + 4 = 4 x 5 = 20 so 20 ÷ 4 = 5 i.e. 20 shared equally between 4 equals 5. Find missing numbers or symbols in a calculation: ☐ ÷ 10 = 3 Represent e.g. using the Singapore bar: 

	Year 3 division	Year 4 division
Mental	If the children know 2/5/10 facts they now need to learn: 3/4/8 facts With corresponding division facts. Recall facts along with counting in steps sizes.  4 x 3 = 3 x 4 12 ÷ 3 = 4 12 ÷ 4 = 3	If the children know multiplication and division facts for: 2/5/10/3/4/8/ they now need to learn: 6/7/9/11 Explore what happens when we divide by 1 and 0. In measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).
Written	Miss West needs 28 paper cups. She has to buy them in packs of 6 How many packs does she have to buy?  GROUPING i.e. 28 ÷ 6 in this context: '28 cups divided into groups of 6'.Modelled using cups then a bead string or bar and recorded on a number line as above. So, 28 ÷ 6 = 4 remainder 4 Also use Grouping ITP:  EQUAL SHARING e.g. There are 28 paper cups to share equally between 6 people. So if everyone has one paper cup, that's one group of 6 paper cups. Demonstrate recording of one group of 6 paper cups on a number line….. Another one for each person etc. So, 6 + 6 + 6 + 6 + 4 = 6 x 4 + 1 = 28 OR so 28 ÷ 6 = 4 r.4 i.e. 28 shared equally between 6 equals 4 remainder 4. **36 ÷ 3 =**  Short multiplication and division rely on mental methods –children should be given short multiplication and division involving 2/3/4/5/6/10 times tables	There are 87 shopping days to Christmas. How many weeks is that? 87 ÷ 7 So, '87 divided into groups of 7' is approximately 70 ÷ 7 = 10. Then begin to divide into 'groups of the divisor' using groups of 10, 5 and 2 wherever possible:  bar model for division  24 ÷ 3 = ? 

	Year 5 division	Year 6 division
Mental	 **Multiplying and dividing whole numbers and decimals by 10, 100 and 1000.** | Thousands | Hundreds | Tens | Ones | /10 (tenths) | /100 (Hundredths) | | | | | | | |	They undertake mental calculations with increasingly large numbers and more complex calculations. Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures. Pupils explore the order of operations using brackets; for example: 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9. Common factors can be related to finding equivalent fractions. Calculate 900 ÷ (45 × 4).
Written	432 ÷ 5 = Estimate: 400 ÷ 5 = 80 Calculate (short division) 432 ÷ 5 becomes 8 6 r 2 5 | 4 3 2 (3) Answer: 86 remainder 2 Estimate: 450 ÷ 15 = 30 Calculate: (Long division) 432 ÷ 15 becomes 2 8 r 12 1 5 | 4 3 2 3 0 0 1 3 2 1 2 0 1 2 Ensure children are able to express remainders either as remainder, fraction or decimal. For example remainder 12 or 12/15 (4/5) or 0.8) Examples with decimals: 37.2 ÷ 8 =	**Short division** 98 ÷ 7 becomes 1 4 (2) 7 | 9 8 Answer: 14 432 ÷ 5 becomes 8 6 r 2 (3) 5 | 4 3 2 Answer: 86 remainder 2 496 ÷ 11 becomes 4 5 r 1 (5) 1 1 | 4 9 6 Answer: $45\frac{1}{11}$ **Long division** 432 ÷ 15 becomes 2 8 r 12 1 5 | 4 3 2 3 0 0 1 3 2 1 2 0 1 2 432 ÷ 15 becomes 2 8 1 5 | 4 3 2 3 0 0 (15×20) 1 3 2 1 2 0 (15×8) 1 2 $\frac{12}{15} = \frac{4}{5}$ 432 ÷ 15 becomes 2 8 · 8 1 5 | 4 3 2 · 0 3 0 ↓ 1 3 2 1 2 0 ↓ 1 2 0 1 2 0 0